

ΛΥΣΕΙΣ ΔΙΑΓΩΝΙΣΜΑΤΟΣ Α ΛΥΚΕΙΟΥ

ΜΑΘΗΜΑΤΙΚΑ 11/3/2017

ΘΕΜΑ 1ο

- A) ΣΧΟΛΙΚΟ βελ.  
 B) ΣΧΟΛΙΚΟ βελ  
 Γ) i) ΣΧΟΛΙΚΟ βελ  
 ii) ΣΧΟΛΙΚΟ βελ  
 Δ) 1. ΞΕΣΤΟ  
 2. ΛΑΘΟΣ  
 3. ΛΑΘΟΣ  
 4. ΛΑΘΟΣ  
 5. ΛΑΘΟΣ

ΘΕΜΑ 2ο

A.1  $27x^4 + x = 0 \Leftrightarrow x(27x^3 + 1) = 0 \Leftrightarrow \begin{cases} x=0 \\ x^3 = -\frac{1}{27} \Leftrightarrow x^3 = (-\frac{1}{3})^3 \Leftrightarrow x = -\frac{1}{3} \end{cases}$

2.  $|x+3| = 2x-2$

Av  $2x-3 \geq 0 \Leftrightarrow x \geq \frac{3}{2}$

$(x+3)^2 = (2x-3)^2 \Leftrightarrow x^2 + 6x + 9 = 4x^2 - 12x + 9 \Leftrightarrow 3x^2 - 18x = 0 \Leftrightarrow 3x(x-6) = 0 \Leftrightarrow \begin{cases} x=0 \\ x=6 \end{cases}$

3.  $|x+1| = |2+3x| \Leftrightarrow (x+1)^2 = (2+3x)^2 \Leftrightarrow x^2 + 2x + 1 = 4 + 12x + 9x^2 \Leftrightarrow 10x = -3 \Leftrightarrow x = -\frac{3}{10}$

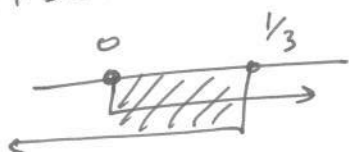
B. i) Θετω  $x^3 = y$  οπότε  $y^2 - 9y + 8 = 0$   
 $\Delta = 81 - 32 = 49$   $y_{1,2} = \frac{9 \pm 7}{2} = \begin{cases} \frac{16}{2} = 8 \\ 1 \end{cases}$

$x^3 = 8 \Leftrightarrow x^3 = 2^3 \Leftrightarrow \boxed{x=2}$

$x^3 = 1 \Leftrightarrow \boxed{x=1}$

ii)  $(x+3)^5 - 16(x+3) = 0 \Leftrightarrow (x+3)((x+3)^4 - 16) = 0 \Leftrightarrow \begin{cases} x+3=0 \Leftrightarrow x=-3 \\ (x+3)^4 = 16 \Leftrightarrow |x+3|=2 \\ \begin{cases} x+3=2 \Leftrightarrow x=-1 \\ x+3=-2 \Leftrightarrow x=-5 \end{cases} \end{cases}$

Γ)  $2(x-1) \leq 3-x \Leftrightarrow 2x+x \leq 3-2 \Leftrightarrow 3x \leq 1 \Leftrightarrow x \leq \frac{1}{3}$   
 $3-x \leq 3x+3 \Leftrightarrow 4x \geq 0 \Leftrightarrow x \geq 0$



$x \in [0, \frac{1}{3}]$

ΘΕΜΑ 30

A) i)  $\left. \begin{aligned} d_4 &= d_1 + 3w \Leftrightarrow 12 = d_1 + 3w \\ d_{10} &= d_1 + 9w \Leftrightarrow 42 = d_1 + 9w \end{aligned} \right\} \rightarrow 42 - 12 = 6w \Leftrightarrow 6w = 30 \Leftrightarrow \boxed{w=5}$

Αρα  $12 = d_1 + 15 \Leftrightarrow \boxed{d_1 = -3}$

ii)  $d_{49} = d_1 + 48w \Leftrightarrow d_{49} = -3 + 48 \cdot 5 \Leftrightarrow d_{49} = 237$

iii)  $d_v = 497 \Leftrightarrow d_1 + (v-1)w = 497 \Leftrightarrow -3 + (v-1)5 = 497 \Leftrightarrow 5(v-1) = 500$   
 $\Leftrightarrow v-1 = 100 \Leftrightarrow \boxed{v=101}$

iv)  $S_{50} = \frac{50}{2} [d_1 + (50-1)w] = 25 \cdot [-3 + 49 \cdot 5] = 25 \cdot 242 = 6.050$

B. i)  $\Delta = (\lambda - 3)^2 - 4(6 - \lambda) = \lambda^2 - 6\lambda + 9 - 24 + 4\lambda = \lambda^2 - 2\lambda - 15$

ηρίζω  $\begin{cases} \alpha > 0 & \delta \text{ως } 1 > 0 \\ \Delta < 0 & \Leftrightarrow \lambda^2 - 2\lambda - 15 < 0 \end{cases}$   $\Delta = 4 + 60 = 64$   
 $\lambda_{1,2} = \frac{2 \pm 8}{2} = \begin{cases} 5 \\ -3 \end{cases}$

$\frac{-3 \quad 5}{+\phi - \phi +}$  αρα  $\lambda \in (-3, 5)$

ii) ηρίζω  $\Delta = 0$  αρα  $\lambda = -3$  ή  $\lambda = 5$

οποτε αν  $\lambda = -3$  τότε  $x^2 - 6x + 9 = 0 \Leftrightarrow (x-3)^2 = 0 \Leftrightarrow x = 3$

αν  $\lambda = 5$  τότε  $x^2 + 2x + 1 = 0 \Leftrightarrow (x+1)^2 = 0 \Leftrightarrow x = -1$

Γ)  $x^2 + 3x + 1 \leq 2x^2 + x - 2 \Leftrightarrow x^2 - 2x - 3 \geq 0$

$\frac{-1 \quad 3}{+\phi - \phi +}$

$\Delta = 4 + 12 = 16$

$x_{1,2} = \frac{2 \pm 4}{2} = \begin{cases} 3 \\ -1 \end{cases}$

$2x^2 + x - 2 \leq x^2 + x + 14 \Leftrightarrow x^2 - 16 \leq 0 \Leftrightarrow x^2 \leq 16$

$|x| \leq 4 \Leftrightarrow -4 \leq x \leq 4$

$\frac{-4 \quad 4}{+\phi - \phi +}$

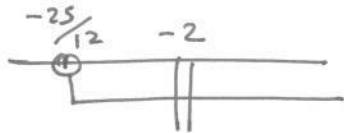


$x \in [-4, -1] \cup [3, 4]$

### ΘΕΜΑ 4ο

1. πρέπει  $\lambda + 2 \neq 0$   
 $\lambda \neq -2$

κ  $\Delta > 0$  οπότε  $(2\lambda + 3)^2 - 4(\lambda + 2)(\lambda - 2) > 0 \Leftrightarrow$   
 $4\lambda^2 + 12\lambda + 9 - 4(\lambda^2 - 4) > 0$   
 $4\lambda^2 + 12\lambda + 9 - 4\lambda^2 + 16 > 0$   
 $12\lambda + 25 > 0 \Leftrightarrow \lambda > -\frac{25}{12}$



Άρα  $\lambda \in \left(-\frac{25}{12}, -2\right) \cup (-2, +\infty)$

2. Αν  $\lambda \neq -2$  τότε  $S = x_1 + x_2 = -\frac{b}{a} = -\frac{-(2\lambda + 3)}{\lambda + 2} = \frac{2\lambda + 3}{\lambda + 2}$   
 $P = x_1 \cdot x_2 = \frac{\gamma}{\delta} = \frac{\lambda - 2}{\lambda + 2}$

3.  $(x_1 + x_2 - 1)^2 + (x_1 \cdot x_2 + 3)^2 = 0 \Leftrightarrow$

$$(S - 1)^2 + (P + 3)^2 = 0 \Leftrightarrow \left(\frac{2\lambda + 3}{\lambda + 2} - 1\right)^2 + \left(\frac{\lambda - 2}{\lambda + 2} + 3\right)^2 = 0$$

οπότε πρέπει  $\begin{cases} \frac{2\lambda + 3}{\lambda + 2} = 1 \Rightarrow 2\lambda + 3 = \lambda + 2 \Rightarrow \lambda = -5 \\ \frac{\lambda - 2}{\lambda + 2} = -3 \Rightarrow \lambda - 2 = -3\lambda - 6 \Rightarrow 2\lambda = -4 \Rightarrow \lambda = -2 \end{cases}$

Επομένως δεν υπάρχει  $\lambda \in \mathbb{R}$  ώστε να ισχύει η παραπάνω σχέση.

4. Αν  $\lambda = -2$  τότε  $|(-2+2)x^2 - (-4+3)x - 4| + \sqrt{x^2 - 5x + 4} + |x^2 - 16| = 0$

$$|x - 4| + \sqrt{x^2 - 5x + 4} + |(x - 4)(x + 4)| = 0 \text{ πρέπει}$$

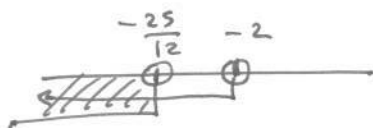
$$x^2 - 5x + 4 = 0 \quad \Delta = 25 - 16 = 9$$
$$x_{1,2} = \frac{5 \pm 3}{2} = \begin{cases} 4 \\ 1 \end{cases}$$

$$\begin{cases} x - 4 = 0 \\ x^2 - 5x + 4 = 0 \Rightarrow \begin{cases} x = 4 \\ x = 1 \end{cases} \\ x^2 - 16 = 0 \Rightarrow \begin{cases} x = 4 \\ x = -4 \end{cases} \end{cases}$$

Άρα  $\boxed{x = 4}$

5. πρέπει  $\Delta < 0 \Leftrightarrow \lambda + 2 < 0 \Leftrightarrow \lambda < -2$

$$\Delta < 0 \Leftrightarrow 12\lambda + 25 < 0 \Leftrightarrow \lambda < -\frac{25}{12}$$



Άρα πρέπει  $\lambda \in \left(-\infty, -\frac{25}{12}\right)$